# DETERMINATION OF THE INSULATING PROPERTIES OF CLOSED SHELLS

V. V. Kharitonov

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The solution of the problem of a suddenly repressurized vacuum chamber is analyzed. The laws of variation of pressure and temperature during the filling process are established. The effect of heat exchange with the ambient medium on the duration of the filling process is indicated, and it is concluded that the insulating properties of shells may be similarly determined.

In a number of areas of technology it is very often necessary to determine the thermal insulation properties of various kinds of closed shells.

Because of the complex configuration and multilayer character of certain shells it is not always possible to use the simplest method of determining the specific heat fluxes through the walls. However, a knowledge of the insulating properties of the shell is extremely necessary in connection with various filling and emptying processes (pumping gasholders, displacing liquids from tanks, repressurizing pressure chambers, etc.), since heat exchange with the ambient medium has an important influence on the duration and other characteristics of the process. As will be shown below, by suddenly repressurizing an evacuated volume it is possible to determine quite simply from the duration of the process the insulating properties of the shell, to obtain correction coefficients for engineering calculations.

We will consider a system consisting of a volume $V_t$ in which the temperature and pressure are equal, respectively, to $T_{t.i}$ and $p_{t.i}$, and a surrounding space at pressure $p_0$ and temperature $T_0$, assuming that $p_{t.i} \ll p_0$ and that in the general case $T_{t.i} \neq T_0$. At a certain moment of time $\tau_0$ a membrane of area $f$ bursts, and within a time $\tau_{fin}$ the volume is filled with gas from the surrounding space.*

On the basis of the law of conservation of energy we can write**

$$dI_0 = dI_t - dQ, \tag{1}$$

where $dI_0 = c_p d(T_0 G_0)$ is the total enthalpy of the gas entering the tank in time $d\tau$; $dI_t = c_p d(T_t G_t)$ is the change in the total enthalpy of the gas in the tank on the same time interval; $dQ = qFd\tau$ is the amount of heat entering the tank from outside or escaping through the surface of the volume F.

Moreover, using the law of conservation of matter, we write

$$dG_0 = dG_t \tag{2}$$

or

$$G_t = G_{t.i} + \int_0^\tau dG_0,$$

where $G_{t.i}$ is the amount of gas in the volume $V_t$ before it starts to fill. This means that the mass of gas in the tank changes as a result of inflow through the opening. Here it should be kept in mind (see Fig. 1) that when $p_{t.i} < p_{cr} = \varepsilon_{cr} p_0$ the inflow will be supercritical [1–3],

$$dG_0 = \frac{\beta f p_0}{\sqrt{RT_0}} d\tau = A_G d\tau, \tag{3}$$

where $A_G$ = const is some constant coefficient equal to the mass flow rate per second

During the subsequent filling process at $p_{cr} < p_t \leq \leq p_0$

$$dG_0 = \frac{f p_0}{\sqrt{RT_0}} \times$$

$$\times \sqrt{2\frac{k}{k-1}\left[\left(\frac{p_t}{p_0}\right)^{\frac{2}{k}} - \left(\frac{p_t}{p_0}\right)^{\frac{k+1}{k}}\right]}\, d\tau. \tag{4}$$

The filling process as a whole can be divided into two stages, the duration of the first stage being $\tau_{cr}$ and that of the second $\tau_{fin} - \tau_{cr}$.

We now rewrite Eq. (1) in the form

$$T_0 dG_0 = T_t dG_t + G_t dT_t - \frac{qF}{c_p} d\tau. \tag{5}$$

Using the equation of state

$$p_t V_t = G_t RT_t. \tag{6}$$

from Eq. (5) we obtain

$$T_0 dG_0 = \frac{V_t}{R} dp_t - \frac{qF}{c_p} d\tau. \tag{7}$$

This equation is integrated by stages.

In the first stage (from $\tau = 0$ to $\tau = \tau_{cr}$ and from $p_{t.i}$ to $p_{cr}$) we have

$$p_t = p_{t.i} + \frac{R}{V_t}\left(T_0 A_G + \frac{qF}{c_p}\right)\tau \tag{8}$$

and

$$\tau_{cr} = \frac{V_t (p_{cr} - p_{t.i})}{A_G RT_0 + \frac{k-1}{k} qF}. \tag{9}$$

To obtain the law of temperature variation in the tank during the first stage we rewrite (5),

$$T_0 dG_0 = T_t dG_t + \frac{p_t V_t}{RT_t} dT_t - \frac{qF}{c_p} d\tau,$$

whence, using (2), (3), (6), and (8), after simple transformations and integration we obtain

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*In what follows we assume that $\tau_0 = 0$.

**We assume that heat losses take a minus sign.

$$T_t = \frac{G_{t.i}\,T_{t.i} + \left(A_G T_0 + \frac{qF}{c_p}\right)\tau}{G_{t.i} + A_G \tau}, \tag{10}$$

where

$$G_{t.i} = \frac{p_{t.i} V_t}{R T_{t.i}}.$$

In the second stage the law of variation of the flow rate (4) is very complicated and integration in quadratures is impossible. Accordingly, we assume a parabolic law of variation of the flow rate, which is quite close to reality, i.e., we assume [4] that

$$dG_0 = \frac{A_G}{1-\varepsilon_{cr}} \sqrt{1 - 2\varepsilon_{cr}(1-\varepsilon) - \varepsilon^2}\, d\tau. \tag{11}$$

Then from (7) with q = 0 we have

$$\tau'_{fin} = \tau_{cr} - \frac{p_0 V_t (1-\varepsilon_{cr})}{A_G R T_0} \times$$

$$\times \left[\arcsin \frac{2ap_t + b}{\sqrt{b^2 - 4ac}}\right]_{p_{t,cr}}^{p_0} \tag{12}$$

and

$$p_t = \varepsilon_{cr}\, p_0 + 0.5 p_0 \sqrt{(2\varepsilon_{cr})^2 - 4(2\varepsilon_{cr} - 1)} \times$$

$$\times \sin \frac{(\tau - \tau_{cr}) A_G R T_0}{(1-\varepsilon_{cr}) p_0 V_t}. \tag{13}$$

For air ($\varepsilon_{cr}$ = 0.528 and $\beta$ = 0.683)

$$\tau'_{fin} = \tau_{cr} + 0.691 \frac{\pi}{2} \frac{V_t}{f\sqrt{RT_0}} \tag{14}$$

and

$$p_t = 0.528 p_0 + 0.471 p_0 \sin \frac{1.445 f \sqrt{RT_0}\,(\tau - \tau_{cr})}{V_t}. \tag{15}$$

In order to take heat transfer into account and obtain a solution in quadratures, we assume a linear law of variation of flow rate in the second stage

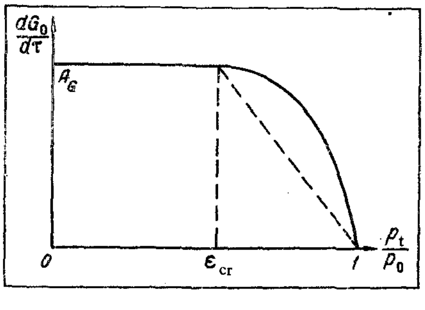

Fig. 1. Flow rate per second as a function of pressure drop.

$$dG_0 = A_G \frac{1-\varepsilon}{1-\varepsilon_{cr}}\, d\tau. \tag{16}$$

Then the solution of Eq. (7) will be

$$\tau''_{fin} = \tau_{cr} + (1-\varepsilon_{cr}) \frac{p_0 V_t}{A_G R T_0} \ln \frac{c_p T_0 A_G + qF}{qF}, \tag{17}$$

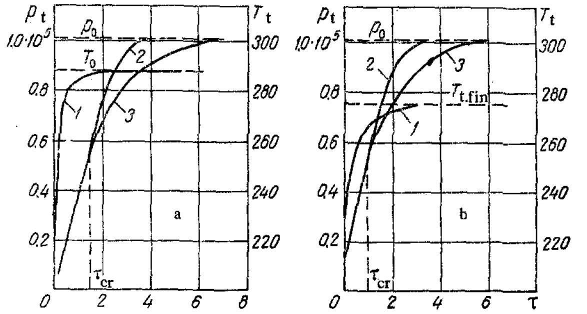

Fig. 2. Variation of the gas parameters as the tank fills from a) $p_{t.i} = 10^{-5}$ torr, b) $p_{t.i}$ = 100 torr: 1) variation of gas temperature $T_t$ (°K); 2 and 3) variation of gas pressure in tank for parabolic and linear laws of flow-rate variation. $p_t$ in N/m², $\tau$ in sec.

$$p_t = p_0 \left[1 + (1-\varepsilon_{cr}) \frac{qF}{c_p T_0 A_G} - \right. \tag{17}$$

$$\left. -(1-\varepsilon_{cr})\left(1 + \frac{qF}{c_p T_0 A_G}\right) \exp(\tau - \tau_{cr}) c\right], \tag{18}$$

where

$$c = \frac{A_G R T_0}{(1-\varepsilon_{cr}) p_0 V_t}.$$

We obtain the law of temperature variation in the second stage if we use equation of state (6), expression (13), and the relation

$$G_t = G_{t.i} + A_G \tau_{cr} + \int_{\tau_{cr}}^{\tau} dG_0.$$

Then

$$T_t = \frac{c p_0 V_t}{A_G R} \left\{c_p T_0 A_G + (1-\varepsilon_{cr}) qF - \right.$$

$$\left. -(1-\varepsilon_{cr})(c_p T_0 A_G + qF) \exp(\tau - \tau_{cr}) c\right\} \times$$

$$\times \left\{c c_p T_0 G_{t.i} + c c_p T_0 A_G \tau_{cr} - c qF (\tau - \tau_{cr}) + \right.$$

$$\left. + (c_p T_0 A_G + qF)[1 - \exp(\tau - \tau_{cr}) c]\right\}^{-1}. \tag{19}$$

The calculations presented in Fig. 2a correspond to the case of a volume $V_t$ = 10 m³ filling with air through an opening $f$ = 0.02 m² from a pressure $p_{t.i}$ = $= 10^{-5}$ mm Hg ($1.33 \cdot 10^{-3}$ N/m²) to $p_0$ = 760 mm Hg ($1.015 \cdot 10^5$ N/m²), when the temperature of the surrounding space $T_0$ = 288° K and $T_{t.i}$ = 223° K (we assume that there is no heat exchange with the ambient medium through the walls).

Clearly, in the first stage (from $\tau$ = 0 to $\tau_{cr}$ = 1.35 sec) the pressure in the tank increases linearly, while the temperature rises almost instantaneously to a value equal to the ambient temperature.

In the second stage the pressure increases more slowly, reaching $p_0$ at $\tau'_{fin}$ = 3.25 sec (parabolic law of flow-rate variation—curve 2) or at $\tau''_{fin} \approx 6$ sec (linear law of flow-rate variation—curve 3). However, the difference in the pressures obtained from the different laws of flow-rate variation is not great and does not exceed 10%.

The change of temperature (curve 1) in the stage is quite negligible.

The results presented in Fig. 2b are for the same vessel but with $p_{t.i}$ = 100 mm Hg ($1.33 \cdot 10^4$ N/m$^2$), the other initial data being similar to the previous case.

Clearly, the variation of the parameters is essentially the same, except that the temperature and pressure curves are flatter. Moreover, since at the higher initial pressure there is a larger mass of gas in the tank, the final temperature is established at a lower level than in the previous case.

The average specific heat flux q through the walls can be determined from the results of full-scale tests using (9) or (18). For this purpose it is necessary only to measure $\tau_{cr}$, the time required for the pressure to reach $p_{cr}$, the total filling time $\tau_{fin}$, and the final temperature established in the tank $T_{t.fin}$.

## NOTATION

V is the volume, m$^3$; T is the temperature, °K; I is the enthalpy, J; Q is the quantity of heat, J; G is the mass of gas, kg; F is the surface of volume, m$^2$; R is the gas constant, J/kg · deg; p is the pressure, N/m$^2$; $\tau$ is the time, sec; $f$ is the clear cross section, m$^2$; $c_p$ is the specific heat, J/kg · deg; q is the specific heat flux, W/m$^2$; $\varepsilon$ is the pressure drop; k is the adiabatic exponent. Subscripts: t) tank; i) initial; fin) final; cr) critical; 0) atmosphere.

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Institute of Heat and Mass
Transfer, AS BSSR, Minsk